Opacity, Credit Rating Shopping and Bias*

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Abstract

We develop a rational expectations model in which the issuer of a financial instrument purchases credit ratings(s) in order to provide useful information to investors and attract investor demand. We examine the nature of a credit-rating equilibrium in a staged game in which an issuer purchases rating(s) sequentially and then decides which ratings to publish. Our analysis emphasizes the importance of opacity about the contacts between the issuer and various rating agencies, leading to potential asymmetric information by investors about the ratings that have been obtained. Too much information is potentially produced and rating agencies are able to extract rents in equilibrium. While the structure of equilibrium forces the disclosure of indicative ratings when the market knows that these have been produced, in some situations investors are unsure as to whether ratings have been obtained that have not been published. Absent disclosure requirements (the opaque case), ratings bias would arise whenever the equilibrium involves publication of fewer ratings than the number of ratings potentially available. However, investors understand the structure of equilibrium and adjust asset pricing to eliminate the potential bias in pricing.

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1 Introduction

In the aftermath of the financial market crisis there has been a considerable spotlight on the accuracy of credit ratings and the potential for upward bias in these, especially in light of the issuer paying for indicative ratings and selectively publishing and disclosing those that would be available for the marketplace to consider in evaluating complex financial instruments.[1] This context raises fundamental questions about the nature of equilibrium. To what degree are the communications between the issuer and rating agency at the stage of obtaining an indicative rating publicly available? Under what conditions do the ratings purchased reflect in equilibrium selective disclosure and bias to which investors would like to adjust? This is central to understanding the nature of credit ratings that are purchased and the implications for asset pricing.

While ratings bias emerges in the literature under a variety of assumptions in which investors react myopically to ratings, it is important to address whether the incentives to shop for ratings and selectively disclose disappear under rationality.[2] For example, is the rationality of investors sufficient to guarantee unbiased ratings? Would the enforcement of mandatory disclosure of indicative ratings guarantee unbiased ratings?

A potential source of ratings bias is the ability of issuers to obtain indicative ratings from rating agencies without being required to disclose these contacts. Of course, disclosure about such indicative ratings could take a variety of forms such as mandatory disclosure of the information provided by the rating agency, disclosure of the contact of a particular rating agency by the issuer in the particular context (e.g., the underlying information might be complex and for which it would be too difficult to mandate disclosure of the fundamental information), or, as has been the case historically, viewing the contact as private. Indeed, the economics of the context could point to the universality of such contacts as when the costs of obtaining indicative ratings are sufficiently low, which often has mirrored the past practice. This discussion suggests a number of policy issues including what types of disclosure should be required and the incentives between the stages of purchasing an indicative rating and publishing the rating. It also emphasizes the importance of the form of equilibrium. Under rational expectations disclosure of contacts with the rating agency is very powerful and can eliminate ratings bias.

We develop a rational expectations framework in which the issuer can help convey infor-

[1] An interesting empirical analysis that documents the potential subjectivity in ratings is Griffin and Tang (2009). The potential for bias in ratings and more specifically, the apparent inverse relationship between ratings standards and the success of a rating organization is illustrated by Lucchetti (2007), who reports that Moody's market share "dropped to 25% from 75% in rating commercial mortgage deals after it increased standards."

[2] One very interesting treatment of ratings under rational expectations is Opp, Opp and Harris (2010). They develop a model to examine information acquisition by a monopolist rating agency in the face of regulatory distortions and asset complexity.

mation to the market by using ratings agencies. We examine the impact of the transparency of the indicative ratings stage. In addition to mandatory disclosure of all indicative ratings we focus upon two disclosure alternatives–one in which all contacts for indicative ratings are transparent and an opaque alternative in which the contacts are not disclosed. If the disclosure of indicative ratings already purchased is costless, then in this equilibrium all purchased ratings are disclosed, implying that ratings shopping and ratings bias do not arise. In the spirit of Akerlof (1970), when publication is costless investors would form an extremely adverse inference when indicative ratings were not disclosed.[3] More generally, if it is common knowledge that a set of indicative ratings have been purchased, then all of these must be disclosed.

In contrast, in the absence of any disclosure requirement about ratings contacts (the opaque case) ratings shopping and ratings bias would arise whenever the equilibrium entails publication of fewer ratings than the number of indicative ratings purchased—as the issuer would then selectively choose which ratings to publish and would choose the highest indicative ratings obtained (e.g., if publishing only one rating, then the issuer would publish the highest rating). Discretionary disclosure arises in equilibrium in the opaque case because the issuer can avoid a completely adverse inference as investors do not know whether ratings are not being disclosed because they were not obtained and therefore unavailable, or because the rating was sufficiently adverse.[4] In the former case, the inference from the absence of a (second) rating would not be adverse.

More specifically, consider a game in which there are two rating agencies and the issuer decides sequentially whether to purchase the second indicative rating after observing the first indicative rating. If the first rating is sufficiently high (i.e., over a relevant threshold), then the issuer would conclude that it doesn't need to expend resources for an additional rating, while if the first rating is lower than the threshold, then the issuer purchases a second indicative rating and then discloses it (but not the first rating) if it is over the same threshold. Notice that only a single rating is disclosed when either rating is sufficiently strong. Otherwise (when both ratings obtained are below the threshold), both ratings are disclosed (the inference would be less favorable if only one rating were disclosed and it were below the threshold). When a single rating is disclosed in this equilibrium, because of the common threshold applied to both

[3] Also, see the related intuition in Grossman (1981) and Milgrom (1981). Furthermore, Verrecchia (1983) examines the case of costly disclosure, demonstrating that disclosure is optimal for realizations at or above a critical level (related ideas in a principal-agent context with costly state verification are developed in Townsend (1979)). The greater the cost the higher is the disclosure threshold and the more selective and discretionary is disclosure. Within Verrecchia's framework public disclosure is optimal for all realizations when it is costless and disclosure is not discretionary (the Akerlof (1970) intuition in his context). By assumption we assume that there are no costs at the publication stage; costs are only incurred when the indicative ratings are obtained.

[4] Indeed, much of the focus of earlier theoretical papers has been on situations in which ratings shopping and selective disclosure do arise in equilibrium (see, for example, Bolton, Freixas and Shaprio (2008), Mathis, McAndrews and Rochet (2009), Sangiorgi, Sokobin and Spatt (2009), and Skreta and Veldkamp (2009)). A notable exception is Opp, Opp and Harris (2010), who also use rational expectations and examine a context with a single credit-rating agency.

rating agencies the investor is unable to distinguish the cases in which this rating was obtained from a single rating agency or from two. Of course, there is rating bias in the scenario when one rating is disclosed. The bias results from the possibility of being in the scenario in which the second rating was over the threshold (and then the first rating was below the threshold).

While we are able to show that an equilibrium emerges without ratings bias under specific conditions (such as transparency of ratings contacts), the opaque case makes clear that the absence of bias is not a robust or universal outcome of our model—even under the assumption of rational expectations. Recent empirical evidence in Kronlund (2011) suggests that ratings shopping has distorted the actual ratings on corporate bonds (ratings are relatively higher for issues that are more likely to experience ratings shopping). Yet interestingly, Kronlund's evidence suggests that investors adjust for this in market pricing, despite the selection and bias in the underlying ratings (yields adjust to the biased ratings). In effect, the market pricing reflects the potential winner's curse associated with the choice of rating agencies (also see discussion in Sangiorgi, Sokobin and Spatt (2009)).

Ratings shopping has had a number of impacts on recent policy debates. For example, the New York State Attorney General's 2008 settlement with the three major rating agencies mandating fees at the indicative ratings stage (though not barring them at the publication stage) attempted to reduce ratings shopping (Office of New York State Attorney General, 2008). The greater the fee at the indicative stage the fewer the ratings that will be obtained potentially and the more limited the scope for selective disclosure and ratings shopping. Additionally, ratings bias undercuts the appropriateness of reliance on ratings for regulatory purposes. Critics of rating agencies have suggested that ratings shopping and the ability of the issuer to select rating agencies is an important conflict of interest that distorts the ratings process.

This draft is organized as follows. Section 2 describes the underlying specification of the model and the disclosure policy in the presence of common knowledge. Section 3 addresses the equilibrium in a transparent market and Section 4 examines the equilibrium in an opaque market. Section 5 concludes.

2 Setup

An *issuer* is endowed with one unit of an asset with random payoff

$$X \sim N(\mu_X, \sigma_X^2).$$

The issuer is risk neutral, and has an exogenous holding cost for the asset equal to V. The issuer can either hold the asset or sell it to the market, which is populated by a continuum (with mass equal to one) of risk-averse investors. Investors have CARA preferences with common absolute risk aversion coefficient r. A riskless asset is traded in perfectly elastic supply with net return normalized to zero. All players have rational expectations and zero discount rates. Ex-ante, the issuer and the investors share the same prior information about the asset value. When the asset is priced according to prior information only, the issuer sells to investors for

$$p_0 = \mu_X - r\sigma_X^2,$$

implying ex-ante gains from trade equal to

$$\Delta \equiv V - r\sigma_X^2.$$

We assume $\Delta > 0$.

The issuer can influence investors' valuation of the asset by conveying information to the market via *rating agencies* (RAs). We assume there are two such agencies, endowed with the same rating technology: each rating agency can produce at a cost H an unbiased noisy signal, or *rating*

$$S_i = X + \varepsilon_i,$$

for $i = 1, 2$, with ε_i uncorrelated with X, i.i.d. and

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2).$$

Hence, the two rating agencies have equivalent, but independent technologies. Each RA maximizes profits by setting the fee c_i at which the issuer can purchase its rating, with the constraint that $c_i \geq H$.

The rating process is as follows. The issuer can approach RA i and purchase its indicative rating at cost c_i; in which case the RA produces the rating S_i which is communicated to the issuer. At this point the issuer owns the rating and can either withhold it or make it public through the rating agency. Only in the latter case would investors observe the rating. A crucial feature of the rating process that plays a major role in our analysis is its degree of transparency. The rating process can be either *transparent* or *opaque,* depending on whether or not the act of purchasing a rating is observable by investors. If the market is opaque, all investors observe is the purchased ratings that the issuer voluntarily decides to publish.

The timing of the model is as follows.

1. RAs simultaneously post fees c_1, c_2. Fees are observed by all players.
2. The issuer shops for ratings. The issuer can shop sequentially, that is, can purchase a first rating, and decide whether to purchase the second rating based on the realization of the first rating.

3. After the purchased ratings are observed, the issuer decides which purchased ratings to disclose (if any).

4. The asset is sold to investors for the price p. The payoff X is realized and consumption takes place.

At any point in the time interval $(1, 4)$, the issuer can decide to stop the game and hold the asset, in which case her payoff equals the risk-neutral valuation of the asset given the information she has acquired up to that point net of the holding cost V. In other words, the issuer participation constraint (IPC) needs to be satisfied not only ex-ante, but also at any interim stage.

We remark that the setup we consider is one in which there is no ex-ante information asymmetry between the issuer and the investors. There are ex-ante gains from trade and information is potentially valuable in that it increases investors' demand. There is no agency problem on the part of the rating agencies: purchased ratings are unbiased and reported truthfully. Investors have rational expectations and understand the incentives behind the issuer's actions. Consequently, the outcomes of the model are not driven by investors' naiveté.

This setup represents a somewhat ideal situation in which many of the concerns regarding the rating industry in recent years have been resolved. Still, from a positive perspective, some questions remain. For example, is investors' rationality sufficient to guarantee unbiased ratings? Will incentives to shop for ratings and to selectively disclose high ratings disappear in this case? Also, from a normative perspective: do ex-ante costs eliminate ratings shopping? If enforced, does mandatory disclosure of purchased ratings guarantee unbiased ratings?

The solution concept implemented is the *perfect Bayesian equilibrium* (PBE). In this rating game, a PBE consists of a strategy profile for the issuer, a price function, and a belief system held by investors such that the strategies are sequentially rational given the belief system and the belief system is consistent given the strategy profile.

2.1 Value of information, surplus and efficiency

The issuer's payoff from selling the asset to investors equals the asset price net of the costs incurred to purchase ratings. The CARA-normal framework implies that, conditional on the issuer disclosing all purchased ratings, the asset price equals the conditional expectation of the fundamental minus a risk discount that is proportional to the conditional variance. Therefore, ex-ante, the issuer expects

$$\begin{aligned} \mathrm{E}(p) &= \mathrm{E}\left[\mathrm{E}(X|I) - r\mathrm{Var}(X|I)\right] \\ &= \mu_X - \underbrace{r\mathrm{E}\left[\mathrm{Var}(X|I)\right]}_{\text{Risk premium}}, \end{aligned} \tag{1}$$

where, informally, I denotes the information set available to investors in equilibrium. Information is therefore valuable, ex-ante, because it reduces the risk premium. Taking into account the cost of information production, H, the *net* value of information, Ω, is defined as

$$\Omega = \max_{n \in \{0,1,2\}} r\left[\sigma_X^2 - \mathrm{Var}\left(X|\{S_i\}_{i=1}^n\right)\right] - n \times H. \tag{2}$$

Then, the *potential surplus* is defined as the sum of the ex-ante gains from trade and the net value of information:

$$\Delta + \Omega.$$

Let Π denote the issuer's ex-ante expected equilibrium profits net of the ex-ante value of the outside option (holding the asset), that is

$$\Pi = \mathrm{E}(p) - \mathrm{E}(\text{costs}) - (\mu_X - V), \tag{3}$$

where E(costs) denotes the expected value of the fees that are going to be paid to RAs during the game. RAs profits equal such costs net of the information production cost H. Then, the *equilibrium surplus* is defined as

$$\Pi + \text{RAs profits}.$$

Therefore, we say that an equilibrium is *efficient* if

$$\Delta + \Omega = \Pi + \text{RAs profits},$$

that is, if *i*) trading takes place, and *ii*) the efficient amount of information is produced and transmitted.

2.2 Disclosure policy with common knowledge

Given the assumed ownership structure of ratings, the issuer always has the option of disclosing information selectively. For example, she could purchase one rating and disclose it only if this rating is good enough, or she could purchase both ratings and disclose only the best of the two and so on. Selective disclosure induces a selection bias in the ratings that are published and results in ratings bias. Skreta and Veldkamp (2009) formalize with intuition under the assumption of naive investors. With rational investors, an obvious question is whether this option is viable in equilibrium. Assume that investors know, or correctly anticipate, which ratings are purchased. This does not mean that investors observe the *realization* of the ratings of course, unless ratings are published. Then, we have the following lemma.

Lemma 1 (Unraveling) *Given common knowledge that there have been n purchased ratings, n ratings are disclosed.*

The idea behind this result is simple and well understood. As a simple illustrative example, let $r = 0$ and assume that in equilibrium the issuer is expected to purchase one rating and to disclose it only if $S_i > \bar{S}$. Then, conditional on no disclosure, the price reflects $\mathrm{E}\left(X | \, S_i \leq \bar{S} \right) \leq \bar{S}$. But for all signal realizations $S_i \in \left(E \left(X | \, S_i \leq \bar{S} \right), \bar{S} \right)$, the issuer would get a better price by disclosing, which yields a contradiction. The unraveling result is undone if there are disclosure costs (e.g., Verrecchia, 1983) or some exogenous source of uncertainty about whether a player has information to disclose (e.g., Shin, 2003). As we do not make any of these assumptions, ratings bias cannot arise from selective disclosure in this framework with common knowledge.

3 Equilibrium in the transparent market

This section describes the equilibrium of the model under the assumption that the rating process is transparent: investors are assumed to observe which ratings are purchased. This will provide a benchmark against which we can compare the model's predictions in the opaque case. The model is solved backwards. First we determine the issuer's strategy for exogenously given costs. An immediate consequence of Lemma 1 is that in the transparent market, each purchased rating will be disclosed. Denote

$$c_l = r \left[\mathrm{Var} \left(X | \, S_i \right) - \mathrm{Var} \left(X | \, S_i, S_{-i} \right) \right]; \qquad c_h = r \left[\sigma_X^2 - \mathrm{Var} \left(X | \, S_i \right) \right], \tag{4}$$

where, because of the joint normality assumption, the conditional variances $\mathrm{Var} \left(X | \, S_i \right)$, $\mathrm{Var} \left(X | \, S_i, S_{-i} \right)$ are constants. The following lemma describes the issuer's decision regarding how many ratings to purchase.

Lemma 2 *For $c \leq c_l$ the issuer purchases both ratings, for $c_l < c \leq c_h$ the issuer purchases only one rating, and for $c > c_h$ no rating is purchased.*

The threshold values in (4) have a very clear economic interpretation. c_h represents the ex-ante marginal value of purchasing the first rating. It equals the reduction in the risk premium (the second term in eq. (1)) the issuer expects from disclosing one rating over disclosing no rating. Similarly, c_l equals the marginal value of the second rating. Ratings are imperfect substitutes in this framework, and the incremental reduction in uncertainty decreases with the number of ratings, implying $c_h > c_l$. The value of ratings is related to the value of information: c_l and c_h are increasing in risk aversion and the variance of the fundamental. In the risk neutral case costly ratings would never be purchased.

3.1 Equilibrium in the fee-setting game

Next, we endogenize the fees as the result of competition between the two rating agencies. We assume that fees are announced simultaneously by the rating agencies at the beginning of the game, and that renegotiation is not possible.

Proposition 1. *The equilibrium in the transparent market is one of the following:*

1. *for $H \leq c_l$, RAs set $c_1 = c_2 = c_l$ and the issuer purchases both ratings;*
2. *for $c_l < H \leq c_h$, RAs set $c_1 = c_2 = H$ and the issuer purchases only one rating;*
3. *for $H > c_h$ no rating is purchased.*

The proof of the Proposition also shows how the IPC is never binding in the case. The following properties of this equilibrium are worth mentioning.

- Equilibrium always exists, and the value of information Ω is maximized. The equilibrium is therefore efficient.
- Depending on the primitives, the issuer could purchase either zero, one or both ratings, but in equilibrium does not condition the decision of purchasing the second rating on the value of the first rating.
- In the risk neutral case (or r sufficiently small), no rating is purchased for $H > 0$.
- The issuer is always better off in equilibrium with RAs than without and strictly better off if at least one rating is purchased in equilibrium, that is, if $\Omega > 0$.

Figure 1 provides a numerical illustration of the equilibrium. The blue and red areas represent issuer's profits Π from eq. (3) and RAs profits respectively. As the equilibrium is efficient, the sum of the two equals potential surplus. Moreover, RAs make positive profits only if $H < c_l$, when both ratings are purchased and RAs are therefore not competing. As $c_l < H \leq c_h$, only one rating is purchased and competition in fees drives profits to zero, so that in this region issuer's profits equal total potential surplus, $\Pi = \Delta + \Omega$. As $H > c_h$, the net value of information, Ω, is equal to zero, so that the asset is sold to investors without acquiring any rating, and issuer's profits equal the gains from trade, $\Pi = \Delta$.

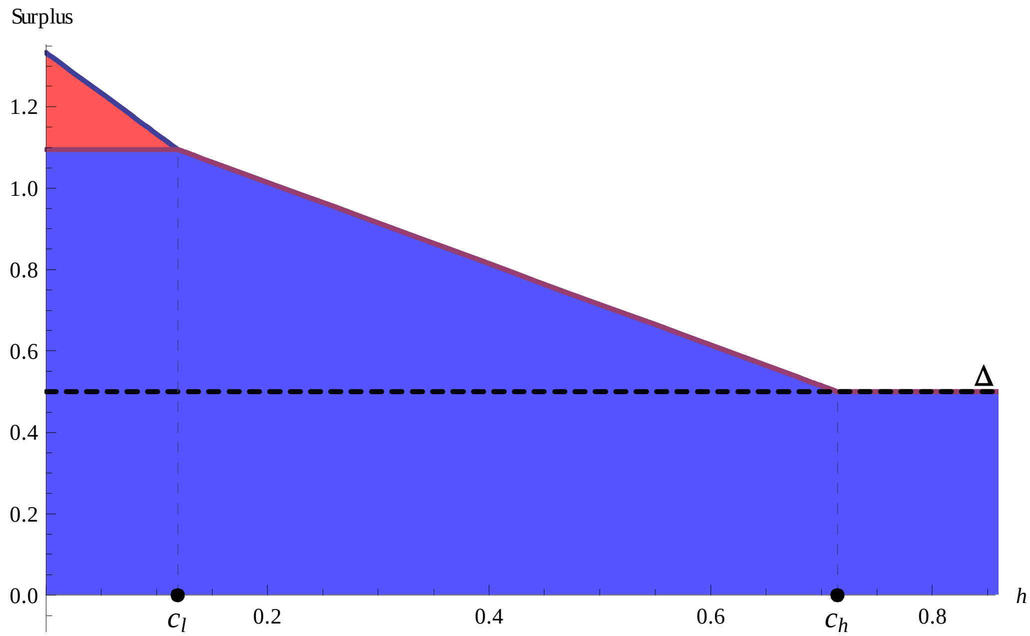

FIGURE 1. Parameter values: $\sigma_X^2 = 5$; $\sigma_\varepsilon^2 = 2$; $r = 0.2$; $V = 1.5$. Red area ($H < c_l$ and Surplus> 0.8): RAs profits. Blue area: issuer's profits Π from eq. (3).

4 Equilibrium in the opaque market

This Section analyzes the case in which investors cannot observe the number of purchased ratings, and highlights the different properties of the resulting equilibrium.

4.1 Equilibrium selective disclosure

In the opaque market, asymmetric information is more severe. In fact, the issuer has private information on which indicative ratings are purchased. Unless both ratings are disclosed, investors do not observe the number of purchased ratings. In this case investors will form beliefs about the number of purchased ratings, and will assign probabilities to the unobservable event in which the issuer is disclosing information selectively. In equilibrium, such probabilities need to be correct, and, by Lemma 1, strictly less than one, or selective disclosure could not be a possibility in the first place. We therefore distinguish two types of equilibria, depending on whether the probability that investors assign (along the equilibrium path) to the issuer disclosing selectively is zero or positive. The following analysis shows how both types of equilibria are possible in the opaque market regime. We remark that source of uncertainty about whether the issuer has information to disclose is not exogenously assumed, but is instead derived as part of the equilibrium with ratings shopping. With some abuse of terminology we will refer to the case in which the equilibrium probability of selective disclosure is zero as a *pure strategy equilibrium*.

4.2 Off-equilibrium beliefs

In a candidate equilibrium, the issuer is expected to follow a given purchasing and disclosure rule. In the opaque market, if investors observe a set of ratings which is off-equilibrium, the deviation could be consistent with the issuer deviating from the purchasing rule or the disclosure rule or both, in a way that makes it impossible to construct off-equilibrium beliefs using Bayes' Rule. For these off-equilibrium information sets we specify investors beliefs explicitly. There are three relevant types of such off-equilibrium information sets.

B1. Investors observe a higher number of ratings than expected. Then we assume investors believe that the number of purchased ratings equals the number of observed ratings.

B2. Investors expect only rating i to be published, and observe instead only rating $-i$. Then we assume investors attach probability one to the path in which the issuer purchased both ratings but only disclosed rating $-i$.

B3. Investors observe a lower number of ratings than expected. Then, we distinguish two cases. Either:

- **a)** given fees c_1, c_2, the observed ratings are consistent with a different equilibrium strategy of the issuer. Then we assume investors adjust their beliefs consistently with the strategy that yields the lowest price among those equilibrium strategies the observed ratings are compatible with; or
- **b)** condition $a)$ is not satisfied, in which case investors attach probability one to the path in which the issuer disclosed information selectively.

The role of refinement (B3-a) is to restore uniqueness of equilibrium whenever harsh off-equilibrium beliefs would sustain multiple equilibria. This refinement is conservative in that, in the presence of multiplicity, the most inefficient equilibria are ruled out.

4.3 Pure strategy equilibria

Here we focus on equilibria in which the issuer discloses all purchased information. Denote with $\Phi(y)$ the CDF and with $f(y)$ the PDF of a Standard Normal valued at y, and define the function

$$g(y) := y\Phi(y) + f(y).$$

Denote

$$\hat{c}_l = \sqrt{\operatorname{Var}(X|S_i) - \operatorname{Var}(X|S_i, S_{-i})}\, g\left(r\sqrt{\operatorname{Var}(X|S_i) - \operatorname{Var}(X|S_i, S_{-i})}\right);$$
$$\hat{c}_h = \sqrt{\sigma_X^2 - \operatorname{Var}(X|S_i)}\, g\left(r\sqrt{\sigma_X^2 - \operatorname{Var}(X|S_i)}\right).$$

From the respective definitions it follows that:

$$\hat{c}_l > c_l; \quad \hat{c}_h > c_h; \quad \hat{c}_h > \hat{c}_l. \tag{5}$$

The following Lemma describes the purchasing decision of the issuer for an exogenous set of fees, and is a counterpart of Lemma 2 for the case of opaque contacts.

Lemma 3. *Assume the IPC is satisfied ex-ante. Then:*

i) $c_i \neq c_j$. *For* $\max\{c_i, c_j\} < \hat{c}_l$, *the issuer purchases both ratings; for* $\min\{c_i, c_j\} < \hat{c}_h$ *and* $\max\{c_i, c_j\} \geq \hat{c}_l$ *the issuer purchases only the cheaper rating, and for* $\min\{c_i, c_j\} \geq \hat{c}_h$ *no rating is purchased.*

ii) $c_i = c_j = c$. *For* $c < \hat{c}_h$ *the issuer purchases both ratings and for* $c \geq \hat{c}_h$ *no rating is purchased.*

A first qualitative difference with Lemma 2 is that in the case of symmetric costs, the equilibrium in which the issuer purchases and discloses only one rating does not arise. The reason is as follows. When the issuer can shop for ratings sequentially in an opaque market, there would always exist a sufficiently bad realization of the first purchased rating that the issuer would find it profitable in expectation to purchase and disclose the second rating. Because of symmetry in the rating costs, she is ex-ante indifferent between purchasing one rating or the other. Therefore investors can only expect one rating, but not a specific one, implying that as long as only one rating is published, they cannot detect any deviation, making the deviation profitable. More generally, interim asymmetric information influences the equilibrium in this context: the issuer cannot refrain from shopping for more ratings and disclosing selectively unless shopping costs (the fees) are sufficiently high. As a result, in order to sell the asset to investors and realize the gains from trade, the issuer might purchase ratings at "unfair" prices, which allows RAs to extract rents. As a consequence, the amount of information produced in equilibrium can be inefficiently high. As an illustration, consider the case with risk neutral investors, $r = 0$, in which the social value of information (2) is zero. In the transparent benchmark, the efficient outcome is implied by Lemma 2: for $r = 0$ and $c > 0$ the issuer sells the asset to investors without acquiring any rating, realizing the ex-ante gains from trade Δ. As $g(0) > 0$, the thresholds $\hat{c}_l$ and $\hat{c}_h$ are strictly positive even with $r = 0$, implying that, surprisingly, ratings retain positive value in equilibrium even if information has no social value.

RAs exploit this to extract more surplus. In this case with $r = 0$, RAs extract surplus from the gains from trade Δ. The outcome is therefore inefficient for all positive cost of information production, $H > 0$.

4.4 Threshold equilibria and ratings bias

Here we describe a situation in which the issuer discloses information selectively with strictly positive probability along the equilibrium path. Our framework provides a natural case in which this may happen. Intuitively, the issuer could purchase a first rating and then decide to shop for the second rating only if the first rating is not good enough. If the second rating happens to be high, then the issuer might want to publish only this one and hide the first, less favorable, rating. Of course, for this option to be viable, it has to be that investors in equilibrium are uncertain as to whether the rating that is published is the result of selective disclosure or not. We formalize this intuition in the following definition. In the following, we relabel ratings so that S_1 is the first rating that ends up being purchased, and S_2 the second one. With symmetry, this is without loss of generality of course, as the issuer is indifferent between which rating to purchase first.

Definition 1. *A threshold $\bar{S}$ and a price function $p_{\bar{S}}(\cdot)$ constitute a* threshold equilibrium *if:*

1. *given the value of the first (randomly selected) rating, S_1, the following strategy is optimal for the issuer:*
   - **i)** *publish the purchased rating and stop if $S_1 \geq \bar{S}$,*
   - **ii)** *purchase the second rating if $S_1 < \bar{S}$ and:*
     - **a)** *publish both ratings if $S_2 < \bar{S}$*
     - **b)** *publish only S_2 if $S_2 \geq \bar{S}$*
2. *the price function $p_{\bar{S}}(\cdot)$ is consistent with the strategy of the issuer.*

Under this conjecture, only two scenarios are possible: either investors observe both ratings with value below $\bar{S}$, or a single rating, which we denote S_p, with value above $\bar{S}$. Whenever both ratings are observed, investors' inference problem is straightforward, and the price of the asset equals

$$p(S_1, S_2) = \mathrm{E}\left(X \,|\, S_1, S_2\right) - r\mathrm{Var}\left(X \,|\, S_i, S_{-i}\right).$$

Whenever a single rating is observed, this is either the result of the first rating being high enough, $S_p = S_1 \geq \bar{S}$, or of the first rating being low and the second being high, that is,

$S_p = S_2 \geq \bar{S}$ and $S_1 < \bar{S}$. In essence, along the conjectured equilibrium path, the distribution of the rating S_p is a mixture of these two truncated distributions. An immediate consequence of this is that the distribution of the published rating S_p is upward biased.[5] Investors' inference problem in this case is more complicated. In fact, the distribution of the fundamental X conditional on the information conveyed by S_p is a mixture of two distributions:

$$\text{prob}\left(X | \, S_p \right) = (1 - q_{(S_p)})\text{prob}\left(X | \, S_p = S_1 \right) + q_{(S_p)}\text{prob}\left(X | \, S_p = S_2 \wedge S_1 < \bar{S} \right),$$

where $q_{(S_p)}$ denotes the posterior probability that the issuer disclosed information selectively, conditional on the value of S_p. As a consequence, investors' end of period wealth, by trading in the asset, is not conditionally normally distributed. As a consequence, we show below, the equilibrium price $p_{\bar{S}}(\cdot)$ is a non-linear function of the the rating. Let $p_{(S_i)}$ denote the price conditional on one rating under full disclosure

$$p(S_i) = \text{E}\left(X | \, S_i \right) - r\text{Var}\left(X | \, S_i \right).$$

Define the following function

$$\Gamma\left(t\right) := \frac{f\left(t\right)}{1 + \Phi(t)}. \tag{6}$$

In the Appendix we show the price $p_{\bar{S}}(S_p)$ to be

$$p_{\bar{S}}(S_p) = p(S_p) - \sqrt{\text{Var}\left(X | \, S_i \right) - \text{Var}\left(X | \, S_i, S_{-i} \right)}\Gamma\left(\frac{\bar{S} - p(S_i)}{\sqrt{\text{Var}\left(S_i | \, S_{-i} \right)}} \right). \tag{7}$$

Intuitively, $p_{\bar{S}}(S_p) < p(S_p)$, as investors adjust pricing in response to the winners' curse implicit in only one rating being published. The following Proposition shows conditions under which such an equilibrium arises.

Proposition 2. *Let $c_i = c_j = c$. Then, there exist functions $v(\cdot)$ and $s(\cdot) < p_0$ and a constant $c_t > \hat{c}_l$, such that $\bar{S} = s(c)$ and the price function given in (7) constitute a threshold equilibrium whenever $c_t < \hat{c}_h$ for all $c \in [c_t, \hat{c}_h)$ and $V \geq v(c)$.*

The Proposition implies that the issuer shops for the second rating only if the first purchased rating happens to be below p_0, the price at which the asset would be sold based on prior information only. The requirement that the cost must be higher than a minimum c_t follows from the fact that the issuer must not find it profitable to deviate from the equilibrium strategy and

[5] More formally, denote with D_1, D_2 the sets of states of the world in which, respectively, only S_1 and only S_2, are published. Let the variable S_p be defined as taking values $S_p(\omega) = S_1$ if $\omega \in D_{1_1}$, $S_p(\omega) = S_2$ if $\omega \in D_2$. Let $D = D_1 \cup D_2$. Then, the bias, b, is defined as

$$b := E\left(S^p - X | \, \omega \in D \right).$$

It is straightforward to verify that $b > 0$.

shop for the second rating if $S_1 \geq \bar{S}$. The minimum requirement on the holding cost V ensures that the IPC is satisfied ax-ante and in any state along the equilibrium path. The condition $c_t < \hat{c}_h$ is a necessary condition for existence of the threshold equilibrium and depends on the primitives of the model. Whenever this is satisfied, the equilibrium strategies in Lemma 3 and Proposition 2 can be combined to illustrate the unique equilibrium strategy of the issuer for a given symmetric value of the costs.

Lemma 4. *Assume the IPC is satisfied ex-ante, $c_t < \hat{c}_h$ and let $c_i = c_j = c$. For $c < c_t$ the issuer purchases and discloses both ratings; for $c \in [c_t, \hat{c}_h)$ the issuer follows the threshold strategy of Definition 1, and and for $c \geq \hat{c}_h$ no rating is purchased.*

We remark that, in the opaque case, whenever equilibrium involves publication of fewer ratings than the number of ratings potentially available (i.e., only one rating), the distribution of these ratings is upward bias.

4.5 Equilibrium in the fee-setting game

In order to describe the outcome of the model in the opaque case, the final step is to determine the equilibrium in the fee-setting game among RAs. For a given equilibrium to arise, a first necessary condition is that the ex-ante IPC is satisfied. The value of the issuer's outside option at the ex-ante stage, $\mu_X - V$, is compared to the ex-ante profits corresponding to a given equilibrium. The relevant quantities are the issuer's expected profits when two ratings are purchased and disclosed

$$\mu_X - r\mathrm{Var}\left(X \middle| S_i, S_{-i} \right) - 2c, \tag{8}$$

and when the issuer follows the strategy of Proposition 2

$$\mathrm{E}\left[p_{s(c)}(S_p) 1_{\omega \in B} + p(S_i, S_{-i}) 1_{\omega \in B^c} \right] - c\left[1 + \Phi\left(\frac{s(c) - \mu_X}{\sigma_s} \right) \right], \tag{9}$$

where B denotes the set of states in which only one rating is published along the path of the threshold equilibrium. The value $v(c)$ in Proposition 2 is such that the ex-ante profits in (9) exceeds the outside option for all $V \geq v(c)$. Define c_2^* as the value of the fees that equates the outside option with (8)

$$c_2^* = \frac{1}{2}\left(\Delta + c_h + c_l \right), \tag{10}$$

and let

$$\tilde{c} = \min\left\{ \hat{c}_t, \hat{c}_h \right\}.$$

The next Proposition describes the equilibrium in the opaque market.

Proposition 3. *The equilibrium in the opaque market is one of the following:*

1. *The equilibrium involves the issuer purchasing and disclosing both ratings if* $H \leq c_2^*$ *and* $H < \tilde{c}$. *Then: for* $c_2^* < \tilde{c}$, *RAs set* $c = c_2^*$, *while for* $c_2^* \geq \tilde{c}$, *any* $c \in [\max\{H, \hat{c}_l\}, \tilde{c})$ *constitutes an equilibrium in fees.*

2. *The equilibrium has the form of Proposition 2 whenever* $c_t < \hat{c}_h$ *and* $H \in [c_t, \hat{c}_h)$ *and* $V \geq v(H)$. *Then RAs set* $c = H$.

3. *If* $H \geq \hat{c}_h$, *no rating is purchased in equilibrium.*

4. *If* $H < \hat{c}_h$ *and neither of the above holds, an equilibrium fails to exist.*

Figure 2 shows the unique outcome of the model in the opaque market for different regions of the primitives. As the figure shows, ceteris paribus, and conditional on ratings being purchased in equilibrium, the threshold equilibrium arises more often for larger values of V and H. High values of the holding cost V could represent situations in which issuers' incentives to sell the asset are high because of high opportunity costs. High values of H might reflect the larger costs it takes to produce a rating for more complex securities. Finally notice that for $H < \hat{c}_h$, when neither conditions in 1-2 of Proposition 3 hold, no equilibrium exists. In this case, the asymmetric information problem induced by ratings shopping is so severe that the market breaks down, and ex-ante valuable trading does not take place.

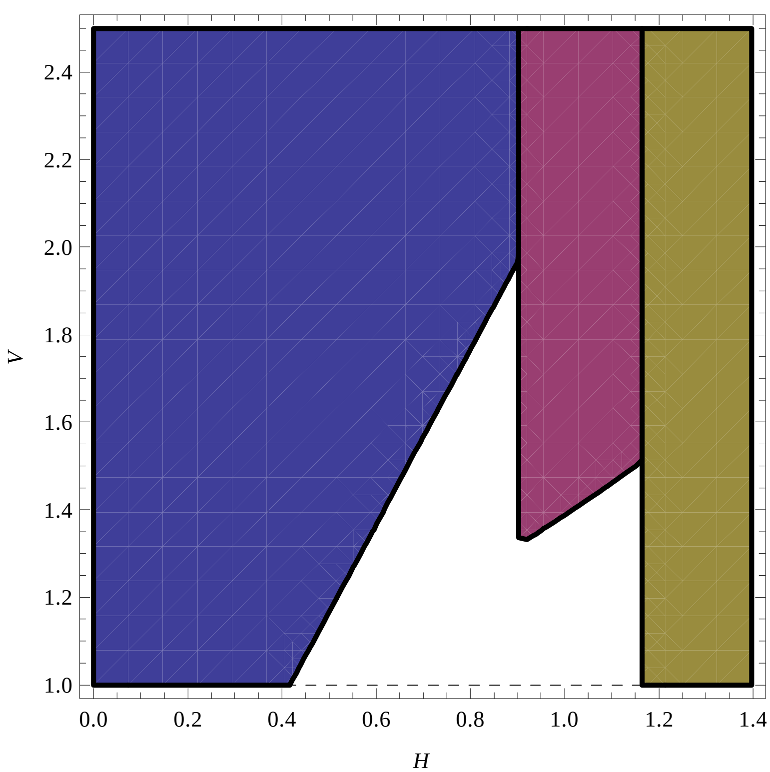

FIGURE 2. Parameter values: $\sigma_X^2 = 5$; $\sigma_\varepsilon^2 = 2$; $r = 0.2$. Blue area ($H < 0.90$): both ratings purchased and disclosed; magenta area ($0.90 \leq H < 1.16$) threshold equilibrium; brown area ($H \geq 1.16$): no ratings purchased. Empty area: no equilibrium.

Figure 3 sets the value of V, and shows the resulting properties of the equilibrium for different values of H. With respect to Figure 1, there are several differences. Equilibrium does not always exist, as shown for $H \in (0.65, 0.9)$ in the figure. When it exists, the equilibrium is inefficient for all $H > c_l$. Inefficiency is measured by the vertical distance between the blue thick line, representing potential surplus, $\Omega + \Delta$, and the sum of RAs and issuer's profits. Inefficiency is the result of inefficient over-production of information. Another difference relates to the distribution of the surplus between the issuer and the RAs. In the equilibrium in which both ratings are purchased, RAs extract all the surplus, including the ex-ante gains from trade. This is because in this numerical example $c_2^* < \hat{c}_h$, so that, from Proposition 3, RAs set $c = c_2^*$ and the IPC is satisfied as an equality. It is only when H is high enough that competition in fees drives RAs profits to zero (in the threshold equilibrium). In this case the issuer's profits are positive, but lower than the ex-ante gains from trade, Δ. Issuer's profits equal Δ only if $H \geq \hat{c}_h$, and no rating is purchased in equilibrium.

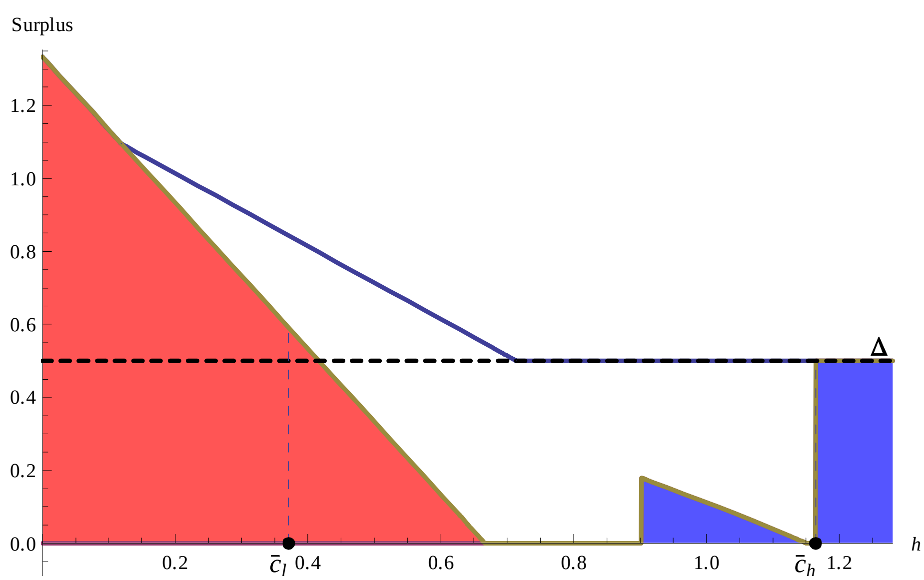

FIGURE 3. Parameter values: $\sigma_X^2 = 5$; $\sigma_\varepsilon^2 = 2$; $r = 0.2$; $V = 1.5$. Blue thick line: potential surplus, $\Omega + \Delta$. Red area: RAs profits. Blue area: issuer's profits Π from eq. (3). Empty area: no equilibrium.

4.6 Endogenous opaqueness

So far we discussed the degree of transparency of the rating process as an exogenous variable. A natural question is which regime would RAs choose if they could. More formally, assume each RA can independently choose the transparency regime as a strategic variable. The contract would therefore specify both the fee at which the rating is sold, and whether the RA would communicate to investors that the rating has been purchased. By Lemma 1, whenever this announcement is credible, as we assume here, purchased ratings are disclosed by the issuer. Therefore, choosing a transparent regime is in fact equivalent to RAs disclosing the purchased rating automatically. Fees and transparency regime are then announced simultaneously by RAs at the beginning of the game and such announcement is observed by all players. The rest of the game is unchanged. Next Proposition establishes whether the equilibrium outcomes described in Propositions 1 and 3 are robust to this extension.

Proposition 4. *When the degree of transparency is endogenously determined, then: i) the opaque market equilibrium of Proposition 3 is unaffected, and ii) the transparent market equilibrium of Proposition 1 is not an equilibrium outcome unless $H \geq \hat{c}_l$.*

Given the previous discussion on the welfare properties of the different regimes, it is not surprising that the opaque regime emerges as an equilibrium while the transparent does not. Enabling the issuer with the option to disclose the rating is beneficial to RAs, and allows them to extract rents in equilibrium.

5 Concluding comments

Our paper uses a model based upon rational expectations to examine conditions under which selective disclosure and ratings bias emerge in equilibrium. We highlight the role of the structure of equilibrium and regulatory policy about disclosure of contacts with rating agencies to purchase indicative ratings. For example, under some conditions requiring the disclosure of the existence of indicative ratings may be equivalent to requiring disclosure of the indicative ratings themselves, eliminating ratings bias in equilibrium. Even mandatory disclosure of contacts about indicative ratings may not be fully effective in eliminating bias and selective disclosure in a setting in which informal discussions between the issuer and the rating agencies can take place prior to the indicative ratings stage. In the absence of requiring disclosure of the contacts about indicative ratings (the opaque analysis), ratings bias can emerge even under rational expectations.$H \geq \hat{c}_l$

The type of analysis we undertake in this paper also is relevant for understanding empirical aspects of credit ratings—especially multiple and split ratings. The information content in ratings reflects not only the ratings selected for publication and disclosure, but also indicative ratings (even though unobservable) that are not selected (also discussed in Sangiorgi, Sokobin and Spatt (2009)). For example, our analysis suggest that at the most favorable rating obtained, the larger the number of these ratings the more favorable the information content as it implies the absence of fewer ratings at lower levels. A similar analysis can apply to split ratings. This also suggest the adverse nature of the absence of ratings (e.g., unrated securities), especially when the costs of ratings are very low.[6] For example, unless the costs are especially high, unrated instruments are likely to reflect especially adverse information. More generally, these types of models highlight the information content of published ratings at various levels.

Our formal analysis does highlight two reasons why issuers might want to publish multiple ratings, even absent regulatory requirements. Because investors are risk averse, additional ratings reduce the required risk premium, offering more precision about the underlying signal.[7] Additionally, to the extent that investors expect the issuer to solicit multiple ratings, absence of publication suggests adverse information, and implies an information discount relative to the issues with public ratings. Indeed, multiple ratings are published with positive probability. In fact, the second motive for multiple ratings is valid even in a risk-neutral setting. These motives tie closely to the "information production hypothesis" and "shopping hypothesis" in Bongaerts, Cremers and Goetzmann (2009).

[6] There is considerable evidence with respect to both multiple ratings and split ratings (e.g., see Bongaerts, Cremers and Goetzmann (2009), Livingston, Naranjo and Zhou (2005) and Mattarocci (2005)).

[7] Also, see Skreta and Veldkamp (2009).

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Appendix A: Notation

Denote with $\Phi\left(y;\mu,\sigma^2\right)$ the CDF and with $f\left(y;\mu,\sigma^2\right)$ the PDF of a Normal variable y with mean μ and variance σ^2, and let

$$\Phi\left(y\right) := \Phi\left(y;0,1\right); \quad f(y) = f(y;0,1).$$

For normal random variables y and z denote with $\mu_{y|z}$ and $\sigma^2_{y|z}$ the first and second moments of y conditional on the realization of z. Furthermore, define the functions $h(\cdot)$ and $g(\cdot)$ by

$$\begin{aligned} h(y) &: = f(y)/\Phi\left(y\right) \\ g(y) &: = y\Phi\left(y\right) + f\left(y\right). \end{aligned}$$

Let $X \sim N(\mu_X, \sigma_X^2)$ and $S_i = X + \varepsilon_i$, with $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ for $i = 1, 2$. Let X, ε_1 and ε_2 be uncorrelated. Then, we have the following standard results

$$\begin{aligned} \mu_{X|S} &= \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + \sigma_\varepsilon^{-2}}(S - \mu_X); \quad \sigma^2_{X|S} = \frac{1}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}, \\ \mu_{X|S_1,S_2} &= \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}\left[(S_1 - \mu_X) + (S_2 - \mu_X)\right]; \quad \sigma^2_{X|2S} = \frac{1}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}. \end{aligned}$$

where

$$\sigma_S^2 = \sigma_x^2 + \sigma_\varepsilon^2.$$

Denoting

$$\mu(y) := \mu_X + \frac{\sigma_X^2}{\sigma_S^2}(y - \mu_X); \quad \mu(y,x) := \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}\left[(y - \mu_X) + (x - \mu_X)\right]$$

then the conditional mean and variance of signal S_{-i} given S_i are

$$\mu_{S-i|S_i} = \mu(S_i); \quad \sigma^2_{S|S} = \sigma^2_{X|S} + \sigma_\varepsilon^2.$$

In this CARA-normal framework, the asset price conditional on, respectively, zero, one and two ratings being published (and full disclosure) are

$$\begin{aligned} p_0 &= \mu_X - r\sigma_X^2 && (11) \\ p(S_i) &= \mu(S_i) - r\sigma^2_{X|S} && (12) \\ p(S_i, S_{-i}) &= \mu(S_i, S_{-i}) - r\sigma^2_{X|sS} && (13) \end{aligned}$$

Appendix B: Proofs of Propositions

Proof of Lemma 2

Denote with $\Pi(n,c)$ issuer's expected profits from purchasing n ratings at cost c. From Lemma 1, purchased ratings are always disclosed, from which if follows that

$$\begin{aligned} \Pi(0,c) &= p_0 = \mu_X - r\sigma_X^2 \\ \Pi(1,c) &= \mathrm{E}\left[p(S_i)\right] - c = \mu_X - r\sigma^2_{X|S} - c \\ \Pi(2,c) &= \mathrm{E}\left[p(S_i, S_{-i})\right] - 2c = \mu_X - r\sigma^2_{X|2S} - 2c \end{aligned}$$

Therefore

$$\begin{aligned}\Pi(2,c) &\geq \Pi(1,c) \quad \Leftrightarrow \quad c \leq r\left(\sigma^2_{X|S}-\sigma^2_{X|2S}\right)\\ \Pi(1,c) &\geq \Pi(0,c) \quad \Leftrightarrow \quad c \leq r\left(\sigma^2_{X}-\sigma^2_{X|S}\right)\\ \Pi(2,c) &\geq \Pi(0,c) \quad \Leftrightarrow \quad c \leq \frac{1}{2}r\left(\sigma^2_{X}-\sigma^2_{X|2S}\right)\end{aligned}$$

For all values of σ_ε and σ_X, we have

$$\sigma^2_{X|S}-\sigma^2_{X|2S} < \frac{1}{2}\left(\sigma^2_X-\sigma^2_{X|2S}\right) < \sigma^2_X-\sigma^2_{X|S}.$$

Therefore, under the convention that, when indifferent, the issuer purchases more ratings, the statement in the Lemma follows immediately.■

Proof of Proposition 1

1. For $H < c_l$, any $c \in [H, c_l)$ is not a Nash equilibrium in the fee-setting game as a RA can deviate to $c+\varepsilon < c_l$ and its rating will be purchased. Similarly, $c > c_l$ is not an equilibrium either as only one rating is purchased, and each RA expects $c/2$. By deviating to $c-\varepsilon$ with $\varepsilon > 0$ sufficiently small, a RA's rating is purchased with probability one, making the deviation profitable. Similar argument holds for $c > c_h$. Instead, at $c = c_l$ both ratings are purchased and no deviation can be profitable as more expensive ratings are not purchased.

2. For $c_l \leq H < c_h$, any $c > H$ cannot be an equilibrium as a deviation to $c-\varepsilon$ with $\varepsilon > 0$ sufficiently small is profitable since the rating would be purchased with probability one. For $c = H$, there is no incentive to deviate as more expensive ratings would not be purchased.

3. Follows trivially from Lemma 1 and $c \geq H$.

Finally, notice that issuer's expected profits with endogenous costs are not smaller than $\Pi(0,c)$ by construction, implying that the IPC is always satisfied.■

Proof of Lemma 3

Denote n_i for $i = 0, 1, 2$ the conjectured equilibrium in which the issuer purchases and discloses n ratings. We identify necessary conditions for n_i to be an equilibrium.

We deal with *case i)* first, in which RAs charge different costs. For simplicity, let $c_1 < c_2$. Consider n_0 first. In an opaque market, the issuer could deviate and purchase rating 1. Given B1, for the deviation not to be profitable, it has to be

$$E\left[\max\left\{p(S_i), p_0\right\}\right] - c_1 \leq p_0, \tag{14}$$

or, equivalently

$$c_1 \geq \int_{p_0}^{\infty}\left[p(S_i)-p_0\right]\frac{1}{\sigma_S}f\left(\frac{S_i-\mu}{\sigma_S}\right)dS_i,$$

which by direct computation gives

$$c_1 \geq \sqrt{\sigma^2_X-\sigma^2_{X|S}}\,g\left(r\sqrt{\sigma^2_X-\sigma^2_{X|S}}\right). \tag{15}$$

The r.h.s of (15) provides the expression for $\hat{c}_h$ in the text. From (15) it follows that $\min\{c_i, c_j\} \geq \hat{c}_h$ is necessary for n_0 to be an equilibrium.[8] Next, consider n_1: in equilibrium the issuer solicits only one

[8]Instead of purchasing only one, the issuer could, upon deviation, purchase two ratings. In this case, it can

rating, which investors anticipate to be rating 1, the cheaper rating. Upon observing the first rating, the issuer could deviate and purchase the second one. In this case, given (B2), it can never be optimal to deviate and publish S_2, unless S_1 is also published. Hence, for the deviation not to be profitable, we must have

$$E\left[\max\left\{p(S_1,S_2),p(S_1)\right\}\middle|\, S_1\right]-c_2\leq p(S_i),\ \text{ for all } S_1\in\mathbb{R}$$

or equivalently,

$$c_2\geq\int_{p(S_1)}^{\infty}\left[p(S_1,S_2)-p(S_1)\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)dS_2, \tag{16}$$

which by direct computation gives

$$c_2\geq\sqrt{\sigma^2_{X|S}-\sigma^2_{X|2S}}\,g\left(r\sqrt{\sigma^2_{X|S}-\sigma^2_{X|2S}}\right). \tag{17}$$

The r.h.s of (17) provides the expression for $\hat{c}_l$ in the text. Since $g(t)-t>0$, $\sqrt{\sigma^2_X-\sigma^2_{X|S}}>\sqrt{\sigma^2_{X|S}-\sigma^2_{X|2S}}$ and $g'(t)=\Phi(t)>0$, it follows that:

$$\hat{c}_l>c_l;\quad \hat{c}_h>c_h;\quad \hat{c}_h>\hat{c}_l. \tag{18}$$

From (18) it follows that $\max\{c_i,c_j\}\geq\hat{c}_l$ is necessary for n_0 to be an equilibrium. The issuer could deviate and publish zero ratings: then (B3-a) applies to n_0 for $\min\{c_i,c_j\}\geq\hat{c}_h$, in which case investors would price the asset p_0, and the deviation would be profitable for S_i low enough. This shows $\min\{c_i,c_j\}<\hat{c}_h$ is also necessary for n_1 to be an equilibrium. In this case harsh beliefs (B3-b) sustain full disclosure.

Finally, n_2. Notice that condition (B3-a) applies to n_0 for $\min\{c_i,c_j\}\geq\hat{c}_h$ and to n_1 for $\max\{c_i,c_j\}\geq\hat{c}_l$. Therefore full disclosure is sustained only if $\min\{c_i,c_j\}<\hat{c}_h$ and $\max\{c_i,c_j\}<\hat{c}_l$, which completes the proof of case i).

In *case ii)* we have $c_i=c_j=c$. We show that n_1 cannot be an equilibrium. With symmetric costs, there is no way for the players to coordinate on a particular rating being purchased. As a consequence, if only one rating is published, investor are not in the position to detect a deviation from the issuer. Hence, for the deviation not to be profitable, we must have

$$E\left[\max\left\{p(S_i,S_{-i}),p(S_i),p(S_j)\right\}\middle|\, S_i\right]-c\leq p(S_i),\ \text{ for all } S_i\in\mathbb{R}.$$

But then, for any c there exist a value of S_i sufficiently low such that the issuer finds it profitable to deviate and solicit the second rating. In fact,

$$\begin{aligned}\lim_{S_i\downarrow-\infty}\frac{E\left[\max\left\{p(S_i,S_{-i}),p(S_i),p(S_j)\right\}\middle|\, S_i\right]-c}{p(S_i)} &= \lim_{S_i\downarrow-\infty}\frac{E\left[p(S_j)\middle|\, S_i\right]}{p(S_i)}\\ &= \frac{\sigma^2_X}{\sigma^2_X+\sigma^2_\varepsilon}.\end{aligned} \tag{19}$$

Given that (19) is the ratio of two negative numbers and is less than one, the issuer will always find it optimal to deviate and purchase the second rating if the value of the first rating is low enough.■

Proof of eq. (7)

First we derive the conditional distribution of the fundamental X given the information contained in the single disclosed rating S_p when the issuer follows the strategy of the threshold equilibrium. Denote

be shown that the condition that guarantees that such a deviation is not profitable is weaker than (15).

$q_{(S_p)}$ the conditional probability of selective disclosure given only one rating is published with value S_p. Let A denote the event that is associated with selective disclosure, and let q =prob$(S_1 < \bar{S} \cap S_2 \geq \bar{S})$ denote the unconditional probability of A. Then,

$$q = \int_{-\infty}^{\bar{S}} \frac{1}{\sigma_s} f\left(\frac{S_1 - \mu_X}{\sigma_s}\right) \Phi\left(\frac{\mu(S_1) - \bar{S}}{\sigma_{S|S}}\right) dS_1$$

Let B denote the event in which only one rating is published with value greater than $\bar{S}$, so that prob(B) =prob$(A \cup S_1 \geq \bar{S})$. Then, we compute the following probabilities

$$\text{prob}(B) = q + \Phi\left(\frac{\mu_X - \bar{S}}{\sigma_S}\right). \tag{20}$$

$$\text{prob}\left(B | A\right) = 1. \tag{21}$$

$$\text{prob}\left(A | B\right) = \frac{q}{q + \Phi\left(\frac{\mu_X - \bar{S}}{\sigma_S}\right)}. \tag{22}$$

$$\text{prob}\left(S_p | A \cap B\right) = \frac{1}{q\sigma_S} f\left(\frac{S_p - \mu}{\sigma_S}\right) \Phi\left(\frac{\bar{S} - \mu(S_p)}{\sigma_{S|S}}\right) 1_{S_p \geq \bar{S}}. \tag{23}$$

$$\text{prob}\left(S_p | B \cap A^C\right) = \frac{1}{\Phi\left(\frac{\mu_X - \bar{S}}{\sigma_S}\right)} \frac{1}{\sigma_S} f\left(\frac{S_p - \mu}{\sigma_S}\right) 1_{S_p \geq \bar{S}}. \tag{24}$$

Denoting with A^C the complement of A, we can use (20)-(24) to compute

$$\begin{aligned} \text{prob}\left(S_p | B\right) &= \text{prob}\left(A | B\right) \times \text{prob}\left(S_p | B \cap A\right) + \text{prob}\left(A^C \middle| B\right) \times \text{prob}\left(S_p | B \cap A^C\right) \\ &= \frac{\frac{1}{\sigma_S} f\left(\frac{S_p - \mu}{\sigma_S}\right)}{q + \Phi\left(\frac{\mu_X - \bar{S}}{\sigma_S}\right)} \left[\Phi\left(\frac{\bar{S} - \mu(S_p)}{\sigma_{S|S}}\right) + 1\right] 1_{S_p \geq \bar{S}}, \end{aligned} \tag{25}$$

and therefore, (25) and (20) imply

$$\begin{aligned} \text{prob}\left(B \cap S_p\right) &= \text{prob}\left(S_p | B\right) \times \text{prob}\left(B\right) \\ &= \frac{1}{\sigma_S} f\left(\frac{S_p - \mu}{\sigma_S}\right) \left[\Phi\left(\frac{\bar{S} - \mu(S_p)}{\sigma_{S|S}}\right) + 1\right] 1_{S_p \geq \bar{S}}. \end{aligned} \tag{26}$$

Using (21), (24) and (26), we can compute the posterior probability of A given one published rating with value equal to S_p:

$$\begin{aligned} q_{(S_p)} &= \frac{\text{prob}\left(A\right) \times \text{prob}\left(B | A\right) \times \text{prob}\left(S_p | A \cap B\right)}{\text{prob}\left(B \cap S_p\right)} \\ &= \frac{\Phi\left(\frac{\bar{S} - \mu(S_p)}{\sigma_{S|S}}\right)}{\left[\Phi\left(\frac{\bar{S} - \mu(S_p)}{\sigma_{S|S}}\right) + 1\right]} 1_{S_p \geq \bar{S}}. \end{aligned} \tag{27}$$

On the equilibrium path $S_p \geq \bar{S}$, and therefore $1_{S_p \geq \bar{S}} = 1$. The conditional density of X given S_p is

$$\begin{aligned} \text{prob}\left(X | B \cap S_p\right) &= \text{prob}\left(A | B \cap S_p\right) \times \text{prob}\left(X | A \cap B \cap S_p\right) + \\ &\quad + \text{prob}\left(A^C \middle| B \cap S_p\right) \times \text{prob}\left(X | A^C \cap B \cap S_p\right) && (28) \\ &= q_{(S_p)} \times \text{prob}\left(X | S_1 \leq \bar{S} \cap S_2 = S_p\right) + \left[1 - q_{(S_p)}\right] \times \text{prob}\left(X | S_1 = S_p\right). && (29) \end{aligned}$$

Conditional on S_p, the fundamental X follows the mixture distribution given in (29). Then, the expected utility of an agent with CARA preferences facing price P conditional on observing S_p and demanding θ units of the asset is[9]

$$\mathrm{E}\left\{-\exp\left[-r\theta(X-P)\right]\middle| S_p, P\right\} = -\exp(r\theta P)\int_{-\infty}^{\infty}\exp\left(-r\theta X\right)\times \operatorname{prob}\left(X\middle| B\cap S_p\right)dX. \tag{30}$$

Solving the integral in (30) using (29), differentiating with respect to θ, setting the derivative to zero, imposing $\theta = 1$, solving for the price P and rearranging we find

$$p_{\bar{S}}(S_p) = p(S_p) - \sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\,\Gamma\left(\frac{\bar{S}-p(S_i)}{\sqrt{\operatorname{Var}\left(S_i\middle| S_{-i}\right)}}\right), \tag{31}$$

where $\Gamma\left(\cdot\right)$ is given in (6).

Proof of Proposition 2

As a first step we derive the function $s(\cdot)$ and the constant c_t as necessary conditions on the equilibrium. We start backwards from stage 2. On the conjectured equilibrium path we have $S_1 < \bar{S}$ and $S_2 \in \mathbb{R}$. We first provide conditions under which publishing S_2 dominates publishing both ratings whenever $S_2 \geq \bar{S}$. We make use the following Lemma:

Lemma A1: *Let $S^* \in R$ be defined by*

$$p_{S^*}(S^*) = p(S^*, S^*) \tag{32}$$

Then: i) S^* exists and is unique; ii) $S^* < p_0$; iii) $p_{\bar{S}}(\bar{S}) \geq p(\bar{S},\bar{S}) \Leftrightarrow \bar{S} \leq S^*$ and iv) $\frac{\partial}{\partial S_2}p_{S^*}(S_2) > \frac{\partial}{\partial S_2}p_{(S_2,S_1)}$.

Proof.

i) Denote

$$t(x,y) := \frac{x-p(y)}{\sigma_{\,S|S}}. \tag{33}$$

From (33) and (12), we have

$$t(z,z) = \frac{p(z,z)-p(z)}{\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}} = \frac{\sigma_\varepsilon(z-p_0)}{\sigma_S\sqrt{\sigma^2_S+\sigma^2_X}};\quad \frac{d}{dz}t(z,z) > 0. \tag{34}$$

Then, if we denote

$$t^* = t(S^*, S^*),$$

(31) and (13) imply

$$p_{S^*}(S^*) = p(S^*,S^*) \Leftrightarrow t^* + \Gamma\left(t^*\right) = 0. \tag{35}$$

Existence of S^* follows from (35) and

$$\lim_{t\uparrow\infty}\Gamma\left(t\right) = \lim_{t\downarrow-\infty}\Gamma\left(t\right) = 0.$$

Given that

$$\Gamma'\left(t\right) = -\Gamma\left(t\right)\frac{\left[g(t)+t\right]}{\Phi\left(t\right)+1};\quad t^*+\Gamma\left(t^*\right) = 0 \Leftrightarrow g(t^*)+t^* = 0 \Leftrightarrow \Gamma'\left(t^*\right) = 0$$

[9] Initial wealth is normalized to zero.

then uniqueness follows from

$$\frac{d}{dt}(t+\Gamma(t))\bigg|_{t=t^*} = 1+\Gamma'(t^*) = 1$$

ii) By (35) and (34) we have

$$p_{S^*}(S^*) = p(S^*,S^*) \Leftrightarrow t(S^*,S^*) < 0 \Leftrightarrow S^* < p_0$$

iii) Follows from i) and

$$\frac{d}{dz}\left(p_z(z) - p(z,z)\right) = -\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\frac{d}{dz}\left(t(z,z) + \Gamma\left(t(z,z)\right)\right) < 0$$

iv) As $\Gamma(t) > 0$ and is unimodal, we have $\Gamma'(t) > 0$ for all $t < t^*$. From (33) it follows that for all $S_p \geq \bar{S}$ and $\bar{S} \leq S^*$ we have

$$t(S^*,S^*) \leq t(\bar{S},\bar{S}) \leq t(\bar{S},S_p).$$

Therefore

$$\frac{\partial}{\partial S_2}\left[p_{\bar{S}}(S_2) - p(S_1,S_2)\right] = \frac{\sigma^2_{X|S} - \sigma^2_{X|2S}}{\sigma^2_\varepsilon}\left[1+\Gamma'\left(t(\bar{S},S_2)\right)\right] > 0$$

■

Lemma A1 implies the following necessary condition for the threshold $\bar{S}$:

$$\bar{S} \leq S^*. \tag{36}$$

Only if (36) is satisfied the issuer, conditional on being in stage 2, finds it optimal to publish S_2 alone for all $S_2 \geq \bar{S}$.

In stage 1, whenever the realization of the first purchased rating is $S_1 \geq \bar{S}$, the issuer might deviate from the conjectured strategy of publishing S_1 by purchasing the second rating. Then, some tedious calculations show the following Lemma

Lemma A3 *For all $S_1 \geq \bar{S}$, we have*

$$\frac{\partial}{\partial S_1}\left[E\left[\max\left\{p_{\bar{S}}(S_1), p_{\bar{S}}(S_2), p(S_1,S_2)\right\}\middle| S_1\right] - p_{\bar{S}}(S_1)\right] < 0 \tag{37}$$

$$E\left[\max\left\{p_{\bar{S}}(S_1), p_{\bar{S}}(S_2), p(S_1,S_2)\right\}\middle| S_1 = \bar{S}\right] - p_{\bar{S}}(\bar{S}) = E\left[\max\left\{p_{\bar{S}}(S_2) - p_{\bar{S}}(\bar{S}), 0\right\}\middle| S_1 = \bar{S}\right] \tag{38}$$

$$\frac{\partial}{\partial \bar{S}}E\left[\max\left\{p_{\bar{S}}(S_2) - p_{\bar{S}}(\bar{S}), 0\right\}\middle| S_1 = \bar{S}\right] < 0. \tag{39}$$

Conditions (37)-(38) imply that the upper bound to the expected payoffs from deviating and purchasing the second rating when $S_1 \geq \bar{S}$ is obtained for $S_1 = \bar{S}$. Therefore it is equal to

$$\Pi_d(\bar{S}) := \int_{\bar{S}}^{\infty}\left[p_{\bar{S}}(S_2) - p_{\bar{S}}(\bar{S})\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(\bar{S})}{\sigma_{S|S}}\right)dS_2. \tag{40}$$

Given c and $\bar{S}$, for the issuer not to deviate it has to be

$$c \geq \Pi_d(\bar{S}). \tag{41}$$

Condition (39) implies that

$$\min_{\bar{S}\leq S^*}\Pi_d(\bar{S}) = \Pi_d(S^*),$$

and therefore the following necessary condition

$$c \geq c_t := \Pi_d(S^*). \tag{42}$$

In fact, only for $c \geq c_t$ there exists some $\bar{S} \leq S^*$ such that (41) holds. From the definition of $\hat{c}_l$ in (16), we can write

$$\hat{c}_l = \int_{p(S^*)}^{\infty} [p(S^*, S_2) - p(S^*)] \frac{1}{\sigma_{S|S}} f\left(\frac{S_2 - \mu(S^*)}{\sigma_{S|S}}\right) dS_2. \tag{43}$$

$S^* < p_0$ implies $p(S^*) \geq S^*$, and the integrand in (40) valued at S^* is easily verified to greater than the integrand in (43), which implies $c_t > \hat{c}_l$.

Notice that, for $c > c_t$, there exists an open interval of threshold values such that (36) is satisfied. To define such interval, for all $c \geq c_t$ let $s(c)$ be the value of the threshold such that (41) holds as an equality, that is,

$$c = \Pi_d(s(c)). \tag{44}$$

Applying the implicit function theorem to (44) and using (39) shows that $s(c)$ is a decreasing function. Also notice $s(c_t) = S^* < p_0$. Then, by construction, for $c > c_t$ any $\bar{S} \in [s(c), S^*]$ is such that both (36) and (42) are satisfied.

Next, we identify additional necessary conditions for the conjectured strategy to be optimal, starting again from stage 2. If both ratings are sufficiently low, the issuer could deviate and publish zero ratings. By (B3-a), the deviation can be profitable only if $c \geq \hat{c}_h$, in which case investors price the asset at p_0. Therefore, it has to be that $c < \hat{c}_h$. Then, consider a given threshold $\bar{S} \in (s(c), S^*]$. For $S_2 < \bar{S}$, if ratings are such that $\hat{S}$ =max$(S_1, S_2) \in [s(c), \bar{S})$ and $\min(S_1, S_2) \leq s(c)$, then the issuer could deviate and publish only the highest rating. Such single published rating is consistent with the threshold equilibrium for all $\bar{S} \in [s(c), \hat{S}]$. Given

$$\frac{\partial}{\partial \bar{S}} p_{\bar{S}}(S_p) = -\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}} \Gamma'\left(t(\bar{S}, S_p)\right) \frac{1}{\sigma_{S|S}} < 0,$$

by (B3-a) investors would price the asset at $p_{s(c)}(\hat{S})$, which by construction is such that $p_{s(c)}(\hat{S}) > p(\hat{S}, s(c))$, implying that the deviation is profitable. Since this applies to all $\bar{S} \in (s(c), S^*]$, we must have $\bar{S} = s(c)$ for harsh beliefs (B3-b) to apply to this deviation, which makes it unfeasible. Similarly, in stage 1, if $\bar{S} = s(c)$ then the issuer cannot profit from deviating and publishing S_1 conditional on $S_1 < \bar{S}$.

It remains to show under which conditions the IPC is satisfied at all interim states as well as ex-ante, which yields the function $v(\cdot)$. From (13) and $V > r\sigma_X^2$, we have that for all S_1, S_2

$$p(S_1, S_2) > \mu(S_1, S_2) - V,$$

and, given $S_2 \geq \bar{S} > S_1$,

$$p_{\bar{S}}(S_2) > p(\bar{S}, S_2) > p(S_1, S_2),$$

implying the IPC is satisfied in all states at stage 2. In stage 1, upon observing $S_1 \geq \bar{S}$, the IPC is satisfied if

$$\mu(S_1) - V \leq p_{\bar{S}(S_1)}. \tag{45}$$

Given

$$\frac{d}{dS_1}\left(\mu(S_1) - V - p_{\bar{S}(S_1)}\right) = -\frac{\sigma^2_{X|S} - \sigma^2_{X|2S}}{\sigma^2_{\varepsilon}} \Gamma'\left(t(\bar{S}, S_2)\right) < 0,$$

then (45) is satisfied for all $S_1 \geq \bar{S}$ if

$$V \geq r\sigma^2_{X|S} + \sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\Gamma\left(t(\bar{S},\bar{S})\right) \tag{46}$$

Upon observing $S_1 < \bar{S}$, the IPC is satisfied if the outside option is not greater than the continuation payoff, that is

$$\mu(S_1) - V \leq -c + \mathrm{E}\left[p_{\bar{S}}(S_2)1_{S_2 \geq \bar{S}} + p(S_1,S_2)1_{S_2<\bar{S}} \middle| S_1\right], \tag{47}$$

which by computation and rearranging gives

$$V \geq c + r\sigma^2_{X|2S} - \int_{\bar{S}}^{\infty}\left[p_{\bar{S}}(S_2) - p(S_1,S_2)\right]\frac{1}{\sigma_{S|S}} f\left(\frac{S_2-\mu(S_2)}{\sigma_{S|S}}\right)dS_2.$$

Given that the r.h.s. of the last expression is maximized as $S_1 \downarrow -\infty$, then (47) is satisfied for all $S_1 < \bar{S}$ if

$$V \geq c + r\sigma^2_{X|2S} \tag{48}$$

Ex-ante, the IPC reads

$$\mu_X - V \leq \mathrm{E}\left[p_{\bar{S}}(S_p)1_{\omega\in B} + p(S_i,S_{-i})1_{\omega\in B^c}\right] - c\left[1+\Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right],$$

where B denotes the set of states in which only one rating is published, and B^c its complement. Computing the expectation and and rearranging gives

$$V \geq r\left[\sigma^2_{X|S}\mathrm{prob}(B) + \sigma^2_{X|2S}(1-\mathrm{prob}(B))\right] + c\left[1+\Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right] + \Xi \tag{49}$$

where $\mathrm{prob}(B)$ is given in (20) and $\Xi \geq 0$ is given by

$$\Xi = \int_{\bar{S}}^{\infty}\left[\Gamma\left(t(\bar{S},S_p)\right) - \Gamma\left(\frac{\bar{S}-\mu(S_p)}{\sigma_{S|S}}\right)\right]\left[1+\Phi\left(\frac{\bar{S}-\mu(S_p)}{\sigma_{S|S}}\right)\right]\frac{1}{\sigma_S} f\left(\frac{S_p-\mu}{\sigma_S}\right)dS_p.$$

Then, the function $v(\cdot)$ is defined to be the r.h.s. of (49) when $\bar{S} = s(c)$. Finally, it is easy to verify that if V is such that (49) holds, then also (46) and (48) hold. That is, if the IPC is satisfied ex-ante, then it is satisfied in any state along the path of the threshold equilibrium.■

Proof of Lemma 4.

By Lemma 3 and Proposition 2, the threshold strategy overlaps with the pure strategy n_2 whenever $c_t < \hat{c}_h$ and $c \in [c_t, \hat{c}_h)$. But then, for these value of the costs, n_2 cannot be an equilibrium: upon observing two ratings, whenever $\hat{S}=\max(S_1,S_2) \geq s(c)$ and $\min(S_1,S_2) < s(c)$, the issuer could deviate and publish only $\hat{S}$. By (B3-a), the single rating $\hat{S}$ is consistent with the issuer following the threshold equilibrium, so that the asset would be priced $p_{s(c)}(\hat{S})$. Given $p_{s(c)}(\hat{S}) > p(\hat{S},\hat{S})$ the deviation is profitable.■

Proof of Proposition 3.

*Two ratings purchased and disclosed***.** From Lemma 4, n_2 can arise only if $H < \min\{\hat{c}_t, \hat{c}_h\}$. Moreover, for the IPC to be satisfied ex-ante we must have $H \leq c_2^*$, where c_2^* defined in (10). In this equilibrium, the equilibrium at the fee-setting stage has the following from. For $c_2^* < \min\{\hat{c}_t,\hat{c}_h\}$, RAs set fees as high as possible, namely $c = c_2^*$ such that the IPC binds with equality and extract all the surplus. RAs have no incentive to deviate as more expensive ratings would violate the IPC and not be purchased. For $c_2^* \geq \min\{\hat{c}_t,\hat{c}_h\}$, there are two cases. If $H \geq \hat{c}_l$, then any $c \in [H, \min\{\hat{c}_t,\hat{c}_h\})$

constitutes an equilibrium in fees: if a RA deviates to a higher rating this would not be purchased by Lemma 3i). Similarly, for $H < \hat{c}_l$, any $c \in [\hat{c}_l, \hat{c}_h)$ constitutes an equilibrium for the same reason.

Threshold equilibrium. Proposition 2 mandates $c_t < \hat{c}_h$, $c \in [c_t, \hat{c}_h)$ and $V \geq v(c)$. Competition in fees imposes an additional set of constraints. In this equilibrium, each RA expects its rating to be purchased with probability $l \in (1/2, 1)$. Therefore, any $c > H$ is not an equilibrium in the fee-setting game. In fact, a RA could deviate to $c' = c - \varepsilon \in (H, c)$: by $c \geq c_t > \hat{c}_l$ and Lemma 3i) its rating would be purchased with probability one. Given $l < 1$ there is always an $\varepsilon > 0$ small enough that $c' > lc$, making the deviation profitable. Instead, $c = H$ is an equilibrium: any deviation to $c' = c + \varepsilon$ with $\varepsilon > 0$ would result in the rating not being purchased, as implied by $c' > c_t > \hat{c}_l$ and Lemma 3i). Therefore we must have $H \in [c_t, \hat{c}_h)$ and $V \geq v(H)$

Zero ratings purchased. By Lemma 4, whenever $c \geq \hat{c}_h$ the equilibrium in which no rating is purchased is viable. Therefore RAs would never charge fees as high as $\hat{c}_h$ unless $H \geq \hat{c}_h$.■

Proof of Proposition 4.

We start by showing that the transparent outcome described in Proposition 1 cannot be an equilibrium for $H < \hat{c}_l$ if RAs can make the contact with the issuer opaque. From Proposition 1, in equilibrium RAs charge $c = c_l$ for $H \leq \hat{c}_l$ and $c = H$ for $c_l < H \leq c_h$. Consider a RA that deviates by switching to opaque contact and fee $c' = c + \varepsilon < \hat{c}_l$. Since $c' < \hat{c}_l$ the issuer cannot commit not to purchase the more expensive rating. Given the IPC is satisfied as a strict inequality in the transparent equilibrium, there always exists an $\varepsilon > 0$ such that the deviation is profitable.

In the opaque case, consider a RA that deviates by switching to a transparent contact and charging $c' \geq c$. The equilibrium described in Proposition 3 is such that $c > \hat{c}_l$. Given $\hat{c}_l > c_l$, it is straightforward to verify that the issuer would always be strictly better off by purchasing only one rating, as opposed to both in the n_2 equilibrium more than one in expectation in the threshold equilibrium. Since the issuer can now credibly commit not to shop for this rating with transparent contact, it is not purchased, so the deviation can never be profitable.■